Calculation of Viscous Drag in Incompressible Flows

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Theme

THE accuracy of calculating the viscous drag of two-dimensional and axisymmetric bodies with the aid of boundary-layer theory is investigated. The boundary-layer parameters were calculated by the Cebeci-Smith (CS) finite-difference method. Drag coefficients of two-dimensional and axisymmetric bodies were calculated from the Squire-Young and Granville formulas, respectively. The calculations were compared with experiments for a wide range of Reynolds numbers. The results show good agreement for airfoils at small angles of attack and for slender bodies of revolution.

Content

Consider the flow around a streamlined body such as an airfoil and assume the boundary layer does not separate from the surface. Starting at the forward stagnation point, there is at first a region of laminar flow, followed by a region (here assumed as a point) of transition from laminar to turbulent flow. In the next region the flow is fully turbulent up to the trailing edge where the boundary layer of the upper surface joins that of the lower surface to form the turbulent wake.

In order to calculate the viscous drag of two-dimensional and axisymmetric bodies, it is necessary to calculate the location of transition (if it is not known a priori) and the boundary-layer growth as accurately as possible to make an accurate drag calculation. The boundary-layer method used in the present study is based on the numerical solution of the boundary-layer equations in their differential form and is due to Cebeci and Smith.[1] Two empirical transition prediction methods were considered in this study, namely those due to Michel[2] and Granville.[3] The method of Michel was primarily used. This allowed direct comparisons with those of a previous study.[4] Michel's method, which is based on a correlation of transition-momentum-thickness Reynolds number, $R_{\theta_{tr}}$, with transition x-Reynolds number, $R_{x_{tr}}$, was approximated by the formula:

$$R_{\theta_{tr}} = 1.174\left(1+\frac{22{,}400}{R_{x_{tr}}}\right)R_{x_{tr}}^{0.46} \quad 0.1 \times 10^6 \leq R_{x_{tr}} \leq 60 \times 10^6 \tag{1}$$

The results obtained from the boundary-layer analysis were used to calculate the total drag coefficient by a formula such as the one given by Granville[3]

$$C_D = \frac{2\theta}{A}\left(\frac{u_e}{u_\infty}\right)^{[(H+2)q+3]/(q+1)} \tag{2}$$

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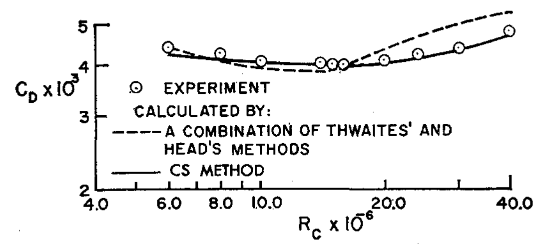

Fig. 1 Calculated and experimental drag coefficients for the NACA $65_{(215)}$-114 airfoil at a lift coefficient $C_l = 0.14$. Transition points were calculated by Michel's method.

where θ, H and u_e/u_∞ are evaluated at the trailing edge of the body.

The above formula reduces to the well known Squire-Young formula for two-dimensional bodies by setting $q = 1$ in which case θ is the momentum thickness and A is the chord. For axisymmetric bodies $q = 7$, θ is the momentum area, and A is a suitable area (such as frontal area). In the calculations reported here, the momentum area was taken to be $\theta = 2\pi r_0 \theta_{2d}$, with θ_{2d} denoting the momentum thickness.[5]

It is clear from Eq. (2) that before the drag coefficient can be calculated, it is necessary to know the velocity ratio, u_e/u_∞. If this ratio is not known experimentally, it can be calculated by inviscid theory. For two-dimensional flows over bodies of finite trailing-edge angle, the theory gives a stagnation point at the trailing edge, thus rendering the Squire-Young formula useless. In order to avoid this difficulty, the velocity distribution has to be extrapolated to the trailing edge. The error in drag arising from this approximation is negligible as shown in Ref. 4.

Figure 1 shows typical results, obtained for a NACA $65_{(215)}$-114 airfoil together with results obtained in an earlier study.[4] The same approach was used in both studies. However, in Ref. 4, the boundary-layer method was based on Thwaites' laminar boundary-layer analysis and Head's turbulent boundary-layer method. The present drag values show a marked improvement over the earlier results which show poor agreement at higher chord-Reynolds numbers.

Figure 2 shows a correlation of calculated and experimental drag values for angles of attack, α, less than 6°. The rms error based on 57 drag values is 2.9%. It is found that the error in calculated drag increases with increasing α. This is shown for three of the many airfoils considered in the insert of Fig. 2. Although the boundary-layer development for unseparated flows can be calculated quite accurately by the CS method, use of the Squire-Young formula introduces errors into the drag calculations. Since this formula is only an approximation, the results show that the approximation becomes increasingly worse with increasing incidence.

The calculation of drag of bodies of revolution is more uncertain than that of two-dimensional bodies. The accuracy of methods available for computing turbulent flows over bodies of revolution, and the validity of the drag formulas are not as well established as those for two-dimensional flows. Also, since the local body geometry enters into the drag calculations, the formulas may not be directly applied at the tail end of bodies with pointed tails. Thus, it becomes necessary to perform the calculations at a point upstream and extrapolate the drag results to the tail end.

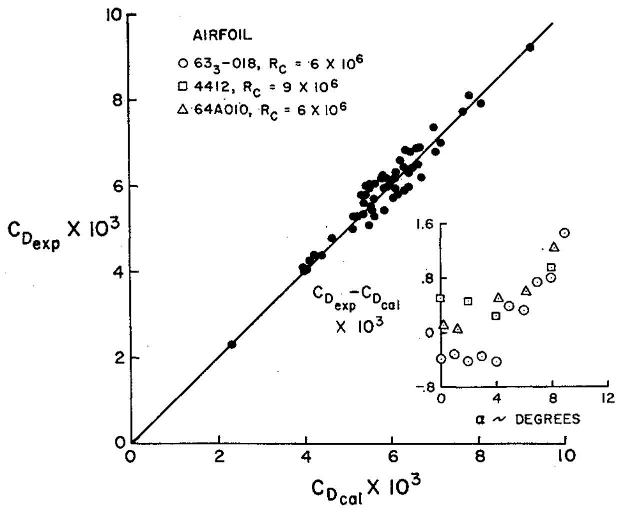

Fig. 2 **Comparison of calculated and experimental results for two-dimensional bodies. The results are shown for 57 drag values at angles of attack, α, less than 6°.**

In addition, if the pressure distribution over the body is unknown, it must be calculated from inviscid theory. In some cases it may also be necessary to apply a viscous correction to this theoretical pressure distribution to account for the presence of the boundary layer. Figure 3 shows a comparison of pressure distributions for a body of fineness ratio 4. The figure shows that viscous effects are small over about 86 percent of the body. Over the remaining portion, however, the pressure distribution with the viscous correction changes significantly from the uncorrected or extrapolated pressure distribution. Application of the formulas at several axial locations near the tail end of the body shows that the formulas are quite sensitive to the choice of a "trailing edge" when the

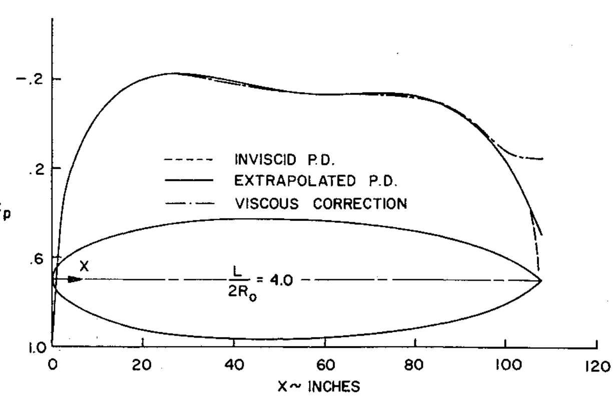

Fig. 3 **Pressure distribution on a body of revolution with and without viscous correction.** $R_L = 4 \times 10^6$.

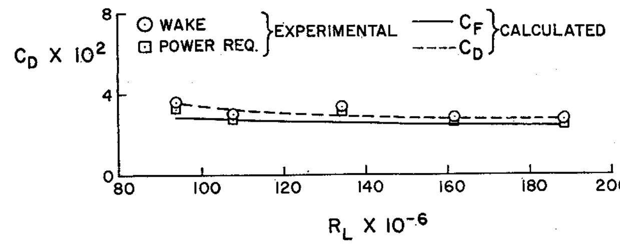

Fig. 4 **Comparison of calculated and experimental results for a 285-ft-long airship. Transition was tripped at 5% from the nose.**

inviscid velocity distribution is used. This sensitivity to the choice of an effective "trailing edge" location seems to disappear when use is made of the pressure distribution with the viscous correction. In that case the total drag values calculated at several axial locations close to the tail end approach a constant value which compares well with experiment. For bodies of larger fineness ratio, good agreement is obtained by using the uncorrected theoretical pressure distribution. A calculation of the skin-friction drag for these bodies shows it to be almost the same as the experimental total drag, thus indicating negligible pressure drag, and justifying the use of uncorrected theoretical pressures.

Figure 4 presents the results for a 285-ft-long airship with a fineness ratio of 4.2 for length-Reynolds numbers from 90 to 200 million. Pressure distribution and boundary-layer measurements for this body were made in full-scale flight. In applying Granville's formula, total drag values were calculated at several axial locations upstream of the tail end and were extrapolated to the tail end. The results in Fig. 4 show that the drag of bodies of revolution at very high Reynolds numbers can be calculated quite well.

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